Statistics

Lecture 5

Feb 19-8:47 AM

SG 10

Intro. to Probabilities:

$E \rightarrow$ desired event (outcome)

$P(E) \rightarrow$ Prob. that event E happens

Acceptable answers:
1) Reduced fraction
2) Round to 3-decimal places
3) Scientific Notation

$$P(E) = \frac{\text{Total \# of all desired outcomes}}{\text{Total \# of all possible outcomes}}$$

A piggy bank has 2 nickels, 3 dimes, and 5 quarters. If we randomly select one coin,

1) $P(\text{Nickel}) = \frac{2 \text{ Nickels}}{10 \text{ Total Coins}} = \boxed{\frac{1}{5}} = \boxed{.2}$

2) $P(\text{Quarter}) = \frac{5 \text{ Quarters}}{10 \text{ Total Coins}} = \boxed{\frac{1}{2}} = \boxed{.5}$

3) $P(\text{Nickel or quarter}) = \frac{7 \text{ Nickel or Quarter}}{10 \text{ Coins}} = \boxed{\frac{7}{10}}$

4) $P(\text{Nickel and quarter}) = \frac{0 \text{ outcomes}}{10 \text{ Coins}} = \boxed{0}$

Do not write $\emptyset$ for Zero.

Jan 15-4:35 PM

I surveyed 80 students. Asked them if they were STEM majors.

	Yes	No	Total
Females	15	30	45
Males	20	15	35
Total	35	45	80

If we randomly select one of them,

1) $P(\text{Female}) = \frac{45}{80} = \boxed{\frac{9}{16}}$

2) $P(\text{Yes}) = \frac{35}{80} = \boxed{\frac{7}{16}}$

3) $P(\text{Female or Yes}) = \frac{65}{80} = \boxed{\frac{13}{16}}$

4) $P(\text{Female and Yes}) = \frac{15}{80} = \boxed{\frac{3}{16}}$

Jan 15-4:44 PM

$E \to$ Desired event

$\bar{E} \to E\text{-bar} \to \text{Not } E \to E\text{-Complement}$

$P(E) + P(\bar{E}) = 1$

$\boxed{P(\bar{E}) = 1 - P(E)}$ Complement Rule

Suppose $P(E) = \frac{3}{16}$, find $P(\bar{E})$.

$P(\bar{E}) = 1 - P(E) = 1 - \frac{3}{16} = \boxed{\frac{13}{16}}$

1 $\boxed{-}$ 3 $\boxed{\div}$ 16 $\boxed{\text{Math}}$ $\boxed{\text{1:▶Frac}}$ $\boxed{\text{Enter}}$

Suppose $P(E) = .025$, find $P(\bar{E})$ in reduced fraction.

$P(\bar{E}) = 1 - P(E) = 1 - .025 = \frac{39}{40}$

1 $\boxed{-}$ .025 $\boxed{\text{Math}}$ $\boxed{\text{1:▶Frac}}$ $\boxed{\text{Enter}}$

Jan 15-4:52 PM

Choose a number from 1 to 25.

1 2 3 4 5 . . . 23 24 25

1) P(Selection is less than 4) = $\frac{3}{25}$

2) P(Selection is at least 20) = $\frac{6}{25}$

3) P(Selection is less than 4 and at least 20)

= $\boxed{0}$

4) P(Selection is less than 4 or at least 20)

= $\frac{9}{25}$

5) P(Selection is multiple of 4) = $\frac{6}{25}$

4, 8, 12, 16, 20, 24

Jan 15-4:57 PM

Some rules & terminologies:

1) $0 \leq P(E) \leq 1$

2) Sum of all probabilities is always 1.

3) $P(E) = 0 \longleftrightarrow$ Impossible event

4) $P(E) = 1 \longleftrightarrow$ Sure event

5) $0 < P(E) \leq .05 \longleftrightarrow$ Rare event

Jan 15-5:04 PM

A standard deck of playing cards has 52 cards,

26 Red, 12 face cards, and 4 aces.

Suppose we draw one card,

1) $P(\text{Red color}) = \frac{26}{52} = \boxed{\frac{1}{2}}$

2) $P(\text{face card}) = \frac{12}{52} = \boxed{\frac{3}{13}}$

3) $P(\overline{\text{Ace}}) = 1 - P(\text{Ace}) = 1 - \frac{4}{52} = \frac{48}{52} = \boxed{\frac{12}{13}}$

4) $P(\text{face or Ace}) = \frac{12+4}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

5) $P(\text{face and Ace}) = \frac{0}{52} = \boxed{0}$

6) $P(\underbrace{\text{Red color or face}}_{\text{There are 6 that are Red \& face.}} \text{ Card}) = \frac{26+12-6}{52} = \frac{32}{52} = \boxed{\frac{8}{13}}$

Jan 15-5:08 PM

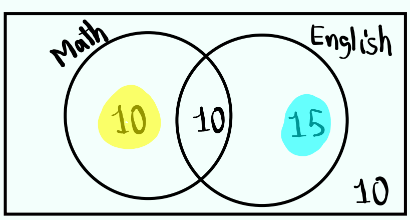
I surveyed 45 students.

20 were taking Math.

25 " " English.

10 " " Both.

1) Construct Venn Diagram



Total = 45

If we randomly select one of them,

P(He/She is taking Math or English, Not both)

$= \frac{10+15}{45} = \frac{25}{45} = \boxed{\frac{5}{9}}$

Jan 15-5:17 PM

If we randomly Select one person,

1) P(He/she has birthday today) $= \dfrac{1}{365}$

2) P(He/she has birthday this month) $= \dfrac{1}{12}$

3) P(He/ she has birthday this week)

$= \dfrac{1}{52}$

SG 10 ✓

Jan 15-5:22 PM

SG 11

Addition Rule

Keyword: OR

Single Action Event

$P(A \text{ or } B) =$

$P(A) + P(B) - P(A \text{ and } B)$

Both

Overlap

ex: $P(A) = .7$, $P(B) = .4$, $P(A \text{ and } B) = .2$

1) $P(\overline{A}) = 1 - P(A) = \boxed{.3}$ 2) $P(\overline{B}) = 1 - P(B) = \boxed{.6}$

3) $P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = \boxed{.8}$

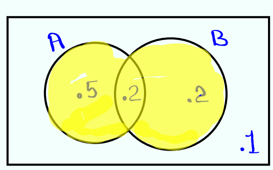
4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Addition Rule

$= .7 + .4 - .2 = \boxed{.9}$

5) $P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .9 = \boxed{.1}$

6) Make Venn Diagram



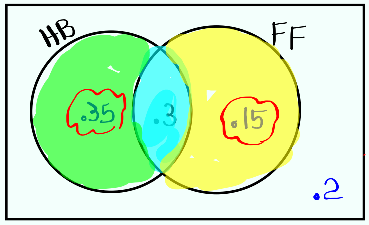
Total = 1 ✓

Jan 15-5:40 PM

$P(HB) = .65$

$P(FF) = .45$

$P(HB \text{ and } FF) = .3$



$.65 - .3 = .35$

$.45 - .3 = .15$

Total $= 1$

$P(\overline{HB}) = 1 - .65 = \boxed{.35}$

$P(\overline{FF}) = 1 - .45 = \boxed{.55}$

$P(\overline{HB \text{ and } FF}) = 1 - .3 = \boxed{.7}$

$P(HB \text{ or } FF)$
$= .65 + .45 - .3 = \boxed{.8}$

$P(\overline{HB \text{ or } FF}) = 1 - .8 = \boxed{.2}$

$P(HB \text{ or } FF\text{, not both})$
$= .35 + .15 = \boxed{.5}$

$P(HB \text{ only}) = .35$

$P(FF \text{ only}) = .15$

Jan 15-5:49 PM

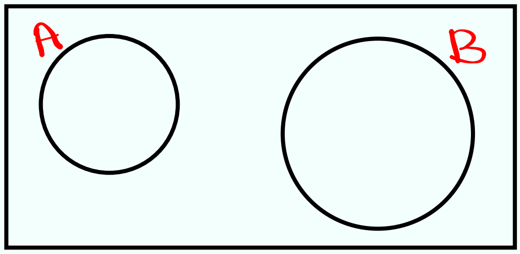
Mutually Exclusive Events

" Disjoint events "

Events that do not happen together

$\boxed{\text{A and B are Mutually Exclusive Events} \iff P(A \text{ and } B) = 0}$

No overlap



Jan 15-5:58 PM

Suppose $P(A) = .72$, $P(B) = .18$,

A and B are M.E.E.

"Disjoint Events"

$P(\overline{A}) = 1 - .72 = \boxed{.28}$

$P(\overline{B}) = 1 - .18 = \boxed{.82}$

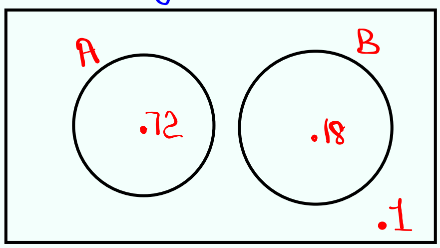
$P(A \text{ and } B) = \boxed{0}$

$P(A \text{ or } B)$:

$P(A) + P(B) - P(A \text{ and } B) =$

$.72 + .18 - 0 = \boxed{.9}$

Draw Venn Diagram



Total = 1

Jan 15-6:01 PM

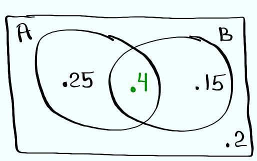
De Morgan's Law:

use it when working with $\overline{A}$ and $\overline{B}$ or $\overline{A}$ or $\overline{B}$

$P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B})$

$P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B})$

Complete the Venn Diagram below:



1) $P(A \text{ and } B)$

$= 1 - [.25 + .15 + .2]$

$= 1 - .6 = \boxed{.4}$

2) Verify Total = 1

3) $P(A) = .25 + .4 = \boxed{.65}$

4) $P(A \text{ only}) = \boxed{.25}$

5) $P(B) = .15 + .4 = \boxed{.55}$

6) $P(B \text{ only}) = \boxed{.15}$

7) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .65 + .55 - .4 = \boxed{.8}$

8) $P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) = 1 - .8 = \boxed{.2}$

De Morgan's Law

9) $P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - .4 = \boxed{.6}$

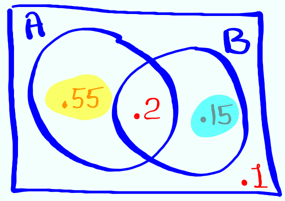
Jan 15-6:07 PM

Given $P(A) = .75$

$P(B) = .35$

$P(A \text{ and } B) = .2$

1) Make Venn Diagram



Total = 1

2) $P(A \text{ or } B) = .75 + .35 - .2 = \boxed{.9}$

3) $P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) = 1 - .9 = \boxed{.1}$

4) $P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - .2 = \boxed{.8}$

5) $P(A \text{ only OR } B \text{ only}) = .55 + .15 = \boxed{.7}$

SG 11 ✓

Jan 15-6:19 PM

SG 12

Odds

I flip a coin 20 times. It landed tails 7 times.

7 tails & 13 $\overline{\text{tails}}$

7 : 13 are odds in favor of landing tails.

Def: odds in favor of event E are

$$a : b$$

# of times E happens

# of times E does not happen.

Odds against Event E are b : a.

Jan 15-6:44 PM

A deck of cards has 40 cards,
25 red, 10 faces, and 3 aces.

$P(\text{Draw a red Card}) = \frac{25}{40} = \frac{5}{8}$

odds in favor of drawing a red Card.

$\# \text{Red} : \# \overline{\text{Red}}$

$25 : 15 \rightarrow \boxed{5:3}$

$25 \boxed{\div} 15$ $\boxed{\text{Math}}$ $\boxed{\text{1:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$

odds against drawing a red Card $\boxed{3:5}$

$P(\text{draw a face Card}) = \frac{10}{40} = \frac{1}{4}$

odds in favor of getting a face Card

$\# \text{face} : \# \overline{\text{face}}$

$10 : 30 \rightarrow 1:3$

odds against $\rightarrow 3:1$

Jan 15-6:49 PM

Odds & Probabilities

Odds in favor of event E are $a:b$.

$$P(E) = \frac{a}{a+b}, \quad P(\overline{E}) = \frac{b}{a+b}$$

ex: Suppose odds for LA Lakers to win the championship this year are $1:39$.

$P(\text{Win}) = \frac{1}{1+39} = \frac{1}{40}$ $\quad P(\overline{\text{Win}}) = \frac{39}{1+39} = \frac{39}{40}$

$1:39$

$\uparrow$

You bet \$1, If they become champ. You get \$39 Net Profit.

Jan 15-6:55 PM

If $P(E)$ is given, odds in favor of event E

are $P(E) : P(\bar{E})$

Always Simplify

Suppose $P(E) = .04$, Find odds in favor of E.

$P(E) : P(\bar{E})$

$.04 : .96 \longrightarrow \boxed{1 : 24}$

$.04$ $\boxed{\div}$ $.96$ $\boxed{\text{Math}}$ $\boxed{\text{1:▶Frac}}$ $\boxed{\text{Enter}}$ $\frac{1}{24}$

Jan 15-7:01 PM

Given $P(E) = .25$

1) $P(\bar{E}) = 1 - .25 = \boxed{.75}$

2) odds in favor of event E.

$.25 : .75 \rightarrow 1 : 3$

3) odds against event E. $3 : 1$

Jan 15-7:04 PM

Use the chart below

x	y
3	8
4	12
5	14
8	20
10	25

1) Find eqn of regression line

$a \approx 2.0$

$b \approx 2.3 \rightarrow \boxed{y \approx 2.0 + 2.3x}$

2) Find linear Correlation Coef.

$r = .995$

3) Find Coef. of determination in whole %. $r^2 \approx 99\%$

99% of Y-values are explained by x-values.

4) Predict Y-Value for $x = 6$

a) Assume $\underbrace{r \text{ is Significant.}}$

Use regression line

$y = 2 + 2.3(6) \approx \boxed{15.8}$

b) Assume $\underbrace{r \text{ is not Significant.}}$

Use $\bar{y} = \boxed{15.8}$

Jan 15-7:07 PM